$\sinh(\theta) = \frac{e^\theta - e^{-\theta}}{2} = \frac{e^{2\theta}-1}{2e^\theta}$ $\cosh(\theta) = \frac{e^\theta + e^{-\theta}}{2} = \frac{e^{2\theta}+1}{2e^\theta}$
$\tanh(\theta) = \frac{\sinh(\theta)}{\cosh(\theta)}$ $\coth(\theta) = \tanh(\theta)^{-1}$
$\text{sech}(\theta) = \cosh(\theta)^{-1}$ $\text{csch}(\theta) = \sinh(\theta)^{-1}$

Ordered Draws with repetition: n^r
Ordered Draws without repetition (Permutation): $\frac{n!}{(n-r)!}$
Unordered Draws without repetition (Combination): $\binom{n}{r} = \frac{n!}{r!(n-r)!}$
Unordered Draws with repetition (multichoose): $\binom{n+r-1}{r}$
$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n^r}{r!}$ $\sum_{r=0}^{n}\binom{n}{r} = 2^n$
$\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$ $\binom{n}{h}\binom{n-h}{k} = \binom{n}{k}\binom{n-k}{h}$
$\binom{n}{k_1,k_2,\ldots,k_r} = \frac{n!}{k_1!k_2!\ldots k_r!}$ $\binom{z}{m}\binom{z}{n} = \sum_{k=0}^{m}\binom{m+n-k}{k,m-k,n-k}\binom{z}{m+n-k}$

$\Gamma(n) = (n-1)!$ $\Gamma(z) = \int_0^\infty e^{-t}t^{z-1}dt$
$\Gamma(1/2) = \sqrt{\pi}$ $\alpha\Gamma(\alpha) = \Gamma(1+\alpha)$
$n! \approx \sqrt{2n\pi}n^n e^{-n}$ $n! = \sqrt{2\pi n}(n/e)^n(1+O(1/n))$
$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$ $B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$
$B(x,y) = \int_0^\infty \frac{t^{x-1}}{(1+t)^{x+y}}dt$

Conditional probability: $P(A|B) = \frac{P(A\cap B)}{P(B)}$
Independent if: $P(A|B) = P(A)$ and $P(B|A) = P(B)$
Which means $P(A\cap B) = P(A)P(B)$
Multiplicative: $P(A\cap B) = P(A)P(B|A) = P(B)P(A|B)$
Associative: $P(A\cap B\cap C) = P((A\cap B)\cap C)$
$= P(A\cap B)P(C|A\cap B) = P(A)P(B|A)P(C|A\cap B)$
Additive: $P(A\cup B) = P(A) + P(B) - P(A\cap B)$
Law of total probability: $B_i \cap B_j = \emptyset\ \forall i \neq j$ and $B_i > 0\ \forall i = 1,2,\ldots,k$ Then $P(A) = \sum_{i=1}^{k} P(A|B_i)P(B_i)$
Bayes' Rule: $P(B_j|A) = \frac{P(A\cap B_j)}{P(A)} = \frac{P(A|B_j)P(B_j)}{\sum_{i=1}^{k}P(A|B_i)P(B_i)}$
$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$
$P(B|A) \propto P(A|B)P(A)$

Expected Value (Discrete): $E(Y) = \sum_y y\cdot p(y) = \mu$
(Continuous): $E(Y) = \int_{-\infty}^{\infty} y\cdot f(y)\,\mathrm{d}y$
$E(a) = a$ for a constant a
$E(X+Y) = E(X) + E(Y)$
$E(ax+b) = a\cdot E(x) + b$
(Discrete) $E(g(y)) = \sum_x g(x)\cdot f(x)$
(Continuous) $E(g(y)) = \int_{-\infty}^{\infty} g(y)\cdot f(y)\,\mathrm{d}y$
$E(XY) = E(X)\cdot E(Y) + \text{cov}(X,Y)$
$E(XY)^2 \leq E(X^2)E(Y^2)$
Variance: $\text{var}(X) = E((X-\mu)^2) = \sigma_X^2 = E(X^2) - E(X)^2$
Std. Dev.: $\sigma_X = +\sqrt{\text{var}(X)}$
(Discrete) $\text{var}(X) = \sum_{x_i}[x_i - \mu]^2 p(x)$
(Continuous) $\text{var}(X) = \int_{-\infty}^{\infty}(y-\mu)^2 f(y)\,\mathrm{d}y$
$\text{var}(aX+b) = a^2\text{var}(X)$
$\text{var}(X+Y) = \text{var}(X) + \text{var}(Y) + 2\text{cov}(X,Y)$
$\text{var}(X-Y) = \text{var}(X) + \text{var}(Y) + 2\text{cov}(X,Y)$
$\text{var}(E(X)) = \frac{\text{var}(X)}{n}$
Covariance: $\text{cov}(Y_1,Y_2) = E((Y_1-\mu_1)(Y_2-\mu_2))$
Independence $\Rightarrow \text{cov}(X,Y) = 0$ but not vice versa
$\text{Cov}(X,Y) = E_Z[\text{Cov}(X,Y|Z)] + \text{Cov}(E[X|Z],E[Y|Z])$

$E[S^r] = \frac{\sigma^r 2^{r/2}\Gamma(\frac{r+n-1}{2})}{(n-1)^{r/2}\Gamma(\frac{n-1}{2})}$

Moment generating function: $M_X(u) = E[e^{ux}]$
$S_n = \sum a_i X_i$, then $M_{S_n} = \prod M_{X_i}(a_i t)$
$E[X^n] = \frac{d^n M_X}{dt^n}(0)$ $\phi_X^{(n)}(0) = i^n E[X^n]$
Cumulant generating function: $K_X(u) = \log M_X(u)$

$f_{X_{(k)}}(x) = n!\frac{F_X(x)^{k-1}}{(k-1)!}\frac{(1-F_X(x))^{n-k}}{(n-k)!}f_X(x)$
$f_{X_{(j)},X_{(k)}}(x,y) =$
$n!\frac{F_X(x)^{j-1}}{(j-1)!}\frac{(F_X(y)-F_X(x))^{k-1-j}}{(k-1-j)!}\frac{(1-F_X(y))^{n-k}}{(n-k)!}f_X(x)f_X(y)$
$f_{X_{(1)},\ldots,X_{(n)}}(x_1,\ldots,x_n) = n!f_X(x_1)\ldots f_X(x_n)$

$X_n \xrightarrow{p} X$ if $\forall\epsilon > 0$ $\Pr(|X_n - X| > \epsilon) \to 0$ as $n\to\infty$
$X_n \xrightarrow{\mathcal{D}} X$ if $F_{X_n}(x) \to F_X(x)$ $\forall x \in C(F_X)$ as $n\to\infty$
$X_n \xrightarrow{a.s.} X$ if $\Pr(\lim_{n\to\infty} X_n \to X) = 1$
$X_n \xrightarrow{a.s.} X \implies X_n \xrightarrow{p} X$ $X_n \xrightarrow{p} X \implies X_n \xrightarrow{\mathcal{D}} X$
If $c \in \mathbb{R}^d$ and $X_n \xrightarrow{\mathcal{D}} c$ then $X_n \xrightarrow{p} c$
$X_n \xrightarrow{p} X$ iff every subseq. has a subsubseq. st $X_{m_j} \to X$ a.s. $j\to\infty$. If $X_n \xrightarrow{a.s.} X$ then $g(X_n) \xrightarrow{a.s.} g(X)$ as $n\to\infty$ for continuous g. Then $X_n \xrightarrow{p} X$, $X_{m_j} \xrightarrow{a.s.} X \implies g(X_{m_j}) \xrightarrow{a.s.} g(X)$

$X_n \xrightarrow{\mathcal{D}} X$
$\equiv E[g(X_n) \to E[g(X)]\ \forall g$ continuous sup on compact set.
$\equiv E[g(X_n)] \to E[g(X)]\ \forall g$ continuous and bounded
$\equiv E[g(X_n)] \to E[g(X)]\ \forall g$ bounded and measurable st $\Pr(X \in C(g)) = 1$

$(X_1,\ldots,X_n)$, iid $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X$
Weak LLN: If $E[X] < \infty$ then $\bar{X}_n \xrightarrow{p} E[X] = \mu$
Strong LLN: $\bar{X}_n \xrightarrow{a.s.} E[X] = \mu \iff E[X] < \infty$
CLT: Y_i iid. $E(Y_i) = \mu$ and $\text{var}(Y_i) = \sigma^2$. $U_n = \frac{\sum_{i=1}^{n} Y_i - n\mu}{\sigma\sqrt{n}}$ $= \frac{\bar{Y}-\mu}{\sigma/\sqrt{n}}$ converges to the standard normal as $n\to\infty$.
That is, $\lim_{n\to\infty} P(U_n \leq u) = \int_{-\infty}^{u}\frac{1}{\sqrt{2\pi}}e^{-t^2/2}\mathrm{d}t$ $\forall u$
$\bar{Y}$ is asymptotically distributed with mean μ and var σ^2/n

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If $X_n \in \mathbb{R}^d, X_n \xrightarrow{\mathcal{D}} X$ if $f: \mathbb{R}^d \to \mathbb{R}^k$ st $\Pr(X \in C(f)) = 1$ then $f(X_n) \xrightarrow{\mathcal{D}} f(X)$
If $X_n \xrightarrow{\mathcal{D}} X$ and $(X_n - Y_n) \xrightarrow{p} 0$ then $Y_n \xrightarrow{\mathcal{D}} X$
If $X_n \in \mathbb{R}^d$, $Y_n \in \mathbb{R}^k$, $X_n \xrightarrow{\mathcal{D}} X$, $Y_n \xrightarrow{\mathcal{D}} c$ then $\binom{X_n}{Y_n} \xrightarrow{\mathcal{D}} \binom{X}{c}$
(Similar for convergence in probability and a.s.)
If $X_n \xrightarrow{p} X$, $Y_n \xrightarrow{p} Y$ then $\binom{X_n}{Y_n} \xrightarrow{\mathcal{D}} \binom{X}{Y}$

Correlation coefficient: $\rho^2 \leq 1$, for $\rho_{Y_1,Y_2} = \frac{\text{cov}(Y_1,Y_2)}{\sigma_1\sigma_2}$
Bias: $E[\delta(g(\theta)) - g(\theta)]$ $MSE(\delta(g(\theta)))$
$= E[(\delta(g(\theta)) - g(\theta))^2] = \text{var}(\delta(g(\theta))) + B(\delta(g(\theta)))^2$
Chebychev's Inequality: $P(|Y-\mu| \geq k\sigma) \leq 1/k^2$ where Y is RV with mean μ, std. dev. σ, and $k \in \mathbb{R}$.
Also, $P(|Y-\mu| \leq k\sigma) \geq 1 - 1/k^2$ or $P(\frac{|Y-\mu|}{\sigma} \geq k)1/k^2$

RVs independent if $F(y_i,y_j) = F_i(y_i)F_j(y_j)$ for each y_i, y_j
MLE $\Rightarrow \hat{\theta}_{mle} = \arg\max_\theta \ln L(\theta|y)$

Conditional distrib.: $f(y_1|y_2) = \frac{f(y_1,y_2)}{f_2(y_2)}$

Hypothesis - Statement about the distribution of a random vector. One on the null, H_0, one on the alternative, H_1. $H_0 \cap H_1 = \emptyset$. The truth is in $H_0 \cup H_1$.

Neyman–Pearson approach: Assume identifiability, Let the test be:

$$\phi(\underset{\sim}{x}) = \begin{cases} 1 & \text{where we reject } H_0 \\ 0 & \text{otherwise} \end{cases}$$

Power function:

$\beta(\theta) = P_\theta(\text{"reject } H_0\text{"}) = P_\theta(X \in C) = E_\theta[\phi(X)]$

Likelihood-Ratio Test:

Let $H_0 : \theta \in \Theta_0$ and $H_1 : \theta \in \Theta_1$, $\Theta_0 \cup \Theta_1 = \Theta$

$\lambda(\underset{\sim}{x}) = \frac{\sup_{\theta \in \Theta_0} L(\theta|\underset{\sim}{x})}{\sup_{\theta \in \Theta_1} L(\theta|\underset{\sim}{x})} = \frac{\sup_{\theta \in \Theta_0} L(\theta|T(\underset{\sim}{x}))}{\sup_{\theta \in \Theta_1} L(\theta|T(\underset{\sim}{x}))}$

$\alpha = \sup_{\theta \in \Theta_0} P_0(X \in C) = \sup_{\theta \in \Theta_0} E_0[\phi(X)]$

$\beta(\theta_0) = \alpha = E[\phi] = \int \phi p_0 d\mu$

$\beta(\theta_1) = E[\phi] =\in \phi p_1 d\mu$

Type I error: $\Pr(x \in C|\theta \in \Theta_0)$

Type II error: $1 - \Pr(x \in C|\theta \in \Theta_1)$

N-P approach: Fix $\Pr(\text{Type I error}) \leq \alpha$ then minimize $\Pr(\text{Type II error})$.

When testing a distribution that is approximately symmetric, consider the equal–tails test:

$\int_{-\infty}^{C_1} f_n(x)dx = \int_{C_2}^{\infty} f_n(x)dx = \alpha/2$

p-value is the smallest α at which you would reject H_0 given some data. The smallest critical region which would lead to rejection.

Monotone LRs: $\theta_1 < \theta 2$ $\frac{p_{\theta_2}(x)}{p_{\theta_1}(x)}$ is nondecreasing in statistic T.

UMP: if $E_\theta \phi^* \geq E_\theta \phi \quad \forall \theta \in \Theta_1$ for all ϕ with level α.

For MLRs: The test $\phi*$ is UMP for $H_0 : \theta \leq \theta_0$ and $H_1 : \theta > \theta_0$ with level defined by $E_{\theta_0}[\phi^*]$.

The power function is nondecreasing.

ϕ^* minimizes the type I ($E_{\theta_1}[\phi^*]$) error for all tests with $E_{\theta_0}[\phi] = \alpha$

$$\phi^*(\underset{\sim}{x}) = \begin{cases} 1 & T(x) > c \\ \gamma & T(x) = c \\ 0 & T(x) < c \end{cases}$$

A test is unbiased if: $\beta(\theta') \geq \beta(\theta'')$ for all $\theta' \in \Theta_1$ and $\theta'' \in \Theta_0$

Asymptotic distribution of LRT:

$2 \log \lambda = 2(\ell_n(\hat{\theta}_n) - \ell_n(\theta_0)) \geq \chi^2_\alpha(r)$ is about size α (asymptotically) where r is the difference in parameter space of null and alt.

Let $A(\theta_0)$ be the acceptance region of a level α test. For each $\underset{\sim}{x}$, let $S(\underset{\sim}{x}) \equiv \{\theta : \underset{\sim}{x} \in A(\theta), \theta \in \Omega\}$ be the $1-\alpha$ confidence level set.

$\theta \in S(\underset{\sim}{x}) \iff \underset{\sim}{x} \in A(\theta)$

Uniformly most accurate unbiased at level $1-\alpha$ (UMAU): minimizes $\Pr_\theta(\theta' \in S(x)) \leq 1-\alpha$ for all $\theta' \neq \theta$ subject to $\Pr_\theta(\theta \in S(x)) \geq 1-\alpha$. Get by inverting UMPU tests.

Family of CDFs $F(t|\theta)$ is stochastically increasing in θ if for $t \in T$, $F(t|\theta)$ is a decreasing function of θ. (For fixed t, think about F as θ changes.)

If X has a continuous CDF, stochastically increasing, let:

$\theta_{u,\alpha}(x) = \sup_{\theta \in \Theta}\{F_X(x|\theta) = \alpha/2\}$

$\theta_{l,\alpha}(x) = \inf_{\theta \in \Theta}\{F_X(x|\theta) = 1-\alpha/2\}$

then $(\theta_{l,\alpha}(X), \theta_{u,\alpha}(X))$ has coverage $1-\alpha$, for all $\theta \in \Theta$

Exponential Families

$p_\theta(x) = C(\theta)\exp\left\{\sum_{j=1}^k Q_j(\theta)T_j(x)\right\} h(x)$

The sufficient statistics form a complete family of distributions.

Have monotone LRs

Look for terms like $\frac{\mu}{\sigma^2}x$ and $-\frac{1}{2\sigma^2}x^2$ for normal

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Bernoulli Distribution

$Y \sim \text{Bernoulli}(\pi) = \text{Binomial}(1, \pi)$

$y \in \{0, 1\} \quad \pi \in [0, 1]$

$p(y|\pi) = \pi^y(1-\pi)^{1-y} \quad \phi(t; \pi) = 1 - \pi + \pi e^{it}$

$E(Y) = \pi$; $\text{var}(Y) = \pi(1-\pi)$

$\kappa_1 = p$; $\kappa_2 = p(1-p)$; $\kappa_{n+1} = p(1-p)\frac{d\kappa_n}{dp}$

Binomial Distribution

$Y \sim \text{Binomial}(n, \pi)$

$y \in \mathbb{Z}_+ \quad n \in \mathbb{N} \quad \pi \in [0, 1]$

$p(y|n, \pi) = \binom{n}{y}\pi^y(1-\pi)^{n-y} \quad \phi(t; \pi) = (1 - \pi + \pi e^{it})^n)$

$E(Y) = n\pi$; $\text{var}(Y) = n\pi(1-\pi)$

$\kappa_{n,binom} = n\kappa_{n,bernoulli}$

Poisson Distribution

$Y \sim \text{Poisson}(\lambda)$

$\lambda > 0$

$p(y|\lambda) = \frac{\lambda^y}{y!}e^{-\lambda} \quad \phi(t; \lambda) = e^{\lambda(e^{it}-1)}$

$E(Y) = \text{var}(Y) = \lambda$

$\text{Poisson}(\lambda) = \lim_{n \to \infty} \text{Binom}(n, \pi = \lambda/n)$

$\kappa_n = \lambda$

Univariate Normal Distribution

$Y \sim Normal(\mu, \sigma^2)$

$y \in \mathbb{R} \quad \mu \in \mathbb{R} \quad \sigma^2 > 0$

$E(Y) = \mu$; $\text{var}(Y) = \sigma^2$

$f(y|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(y-\mu)^2}{2\sigma^2}} \quad \phi(t; \mu, \sigma^2) = e^{it\mu - \sigma^2 t^2/2}$

$\kappa_1 = \mu$; $\kappa_2 = \sigma^2$; $\kappa_n = 0$

Multivariate Normal Distribution

$X \sim N_p(\boldsymbol{\mu}, \Sigma)$

$f(x; \boldsymbol{\mu}, \Sigma) = \left(\frac{1}{\sqrt{2\pi}}\right)^p \frac{1}{|\Sigma|^{1/2}}\exp\left\{-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^T\Sigma^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right\}$

$\Sigma = \text{diag}(d_1, \ldots, d_n)$ iff mutually independent.

$X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim N_p\left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right)$ independent if $\Sigma_{12} = \Sigma_{21} = 0$

Uniform Distribution

$Y \sim \text{Uniform}(\alpha, \beta)$

$y \in [\alpha, \beta]$

$f(y|\alpha, \beta) = \frac{1}{\beta - \alpha} \quad \phi(t; \alpha, \beta) = \frac{e^{it\beta} - e^{it\alpha}}{it(\beta - \alpha)}$

$E(Y) = \frac{\alpha+\beta}{2}$; $\text{var}(Y) = \frac{(\beta-\alpha)^2}{12}$

$\text{Unif}(-a, a) \to k_i = 0$ for i odd; $\kappa_0 = 0$; $\kappa_2 = a^2/3$

$\kappa_4 = \frac{a^4}{5} - 3\left(\frac{a^2}{3}\right)^2$; $\kappa_6 = \frac{a^6}{7} - 15\frac{a^4}{5}\frac{a^2}{3} + 30\left(\frac{a^2}{3}\right)^3$

Gamma Distribution

$Y \sim \text{Gamma}(\alpha, \beta)$

$y \in (0, \infty)$

Shape $\alpha > 0$, inverse scale $\beta > 0$ (i.e. $\beta = 1/\theta$)

$f(y|\alpha,\beta) = \frac{\beta^\alpha y^{\alpha-1}}{\Gamma(\alpha)} e^{-\beta y} \quad \phi(t;\alpha,\theta) = (1 - it\theta)^{-\alpha}$

$E(Y) = \alpha/\beta = \alpha\theta$; $\text{var}(Y) = \alpha/\beta^2 = \alpha\theta^2$

$\text{Gamma}(k, \frac{1}{\lambda} = \text{Erlang}(k, \lambda)$

$\kappa_r = \alpha\Gamma(r)$

Beta Distribution

$Y \sim \text{Beta}(\alpha, \beta)$

$y \in [0, 1]$ Shape parameters: $\alpha, \beta > 0$

pdf: $f(y|\alpha,\beta) = \frac{y^{\alpha-1}(1-y)^{\beta-1}}{B(\alpha,\beta)} = \frac{y^{\alpha-1}(1-y)^{\beta-1}}{\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\,\mathrm{d}t}$

$E(Y) = \frac{\alpha}{\alpha+\beta}$; $\text{var}(Y) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

Student's t Distribution

$Y \sim t(n)$

$y \in \mathbb{R}$ $n \in \mathbb{N}$ is the degrees of freedom.

$f(y|n) = \frac{1}{\sqrt{n}B(1/2, n/2)}(1 + \frac{t^2}{n})^{-\frac{n+1}{2}}$

$E(Y) = 0$ for $n > 1$; $\text{var}(Y) = \frac{n}{n-2}$ for $n > 2$

$t(n-1) = \frac{\bar{x}-\mu}{s/\sqrt{n}}$ when $X \sim N(\mu, \sigma^2)$

$Z \sim N(0,1)$ and $V \sim \chi^2_{n-1}$ then $\frac{Z}{\sqrt{V/(n-1)}} \sim t_{n-1}$

Exponential Distribution

$Y \sim \text{Exp}(\lambda)$

$y \in [0, \infty)$, $\lambda > 0$

$f(y|\lambda) = \lambda e^{-\lambda y} \quad \phi(t;\lambda) = (1 - it/\lambda)^{-1}$

$E[Y] = 1/\lambda \quad \text{var}(Y) = 1/\lambda^2$

$\sum_{i=1}^n \text{Exp}(\lambda) = \text{Erlang}(n, \lambda)$

$k\text{Exp}(\lambda) = \text{Exp}(\frac{\lambda}{k})$

Minimum of n Exponentials: $\text{Exp}(n\lambda)$

$\text{Exp}(\lambda) = \text{Gamma}(1, 1/\lambda)$

$\lfloor \text{Exp}(\lambda) \rfloor = \text{Geometric}(1 - e^{-\lambda})$

$\kappa_r = \lambda^{-r}(r-1)!$

Erlang Distribution

$X \sim \text{Erlang}(k, \lambda)$

$x, \lambda \in [0, \infty)$

$f(x|k,\lambda) = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}$

$E[X] = \frac{k}{\lambda} \quad \text{var}(X) = \frac{k}{\lambda^2}$

Pareto Distribution

$X \sim \text{Pareto}(\nu, \theta)$

$\theta, \nu > 0 \quad x > \nu$

$f(x;\theta,\nu) = \frac{\theta\nu^\theta}{x^{\nu+1}} 1(x \geq \nu)$

$E[X] = \begin{cases} \infty & \theta \leq 1 \\ \frac{\theta\nu}{\theta-1} & \theta > 1 \end{cases} \quad \text{var}(X) = \begin{cases} \infty & \theta \in (1,2] \\ \frac{\nu^2\theta}{(\theta-1)^2(\theta-2)} & \theta > 2 \end{cases}$

$\log(\text{Pareto}(\nu,\theta)/\nu) = \text{Exp}(\theta)$

Rayleigh Distribution

$X \sim \text{Rayleigh}(\sigma)$

$\sigma > 0 \quad x \in [0, \infty)$

$f(x;\sigma) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2}$

$E[X] = \sigma\sqrt{\frac{\pi}{2}} \quad \text{var}(X) = \frac{4-\pi}{2}\sigma^2$

$(\text{Rayleigh})^2 = \chi^2_2$

Chi-squared Distribution

$Y \sim \chi^2(n)$

$y \in [0, \infty) \quad n \in \mathbb{N}$

$f(y|n) = \frac{1}{2^{n/2}\Gamma(n/2)} y^{n/2-1} e^{-y/2} \quad \phi(t;n) = (1 - 2it)^{-n/2}$

$E[Y] = n \quad \text{var}(Y) = 2n$

$\chi^2(n) + \chi^2(k) = \chi^2(n+k)$ if iid

$\chi^2(2) = \text{Exp}(\frac{1}{2})$

$(n-1)\frac{s^2}{\sigma^2} \sim \chi^2_{n-1}$

$\kappa_r = 2^{r-1}(r-1)!n$

Negative Binomial Distribution

$Y \sim NB(r, p)$

$y \in \{0, 1, 2, \dots\} \quad r > 0 \quad p \in (0,1)$

$p(y|r,p) = \binom{y+r-1}{y}(1-p)^r p^y \quad \phi(t;r,p) = \left(\frac{1-p}{1-pe^{it}}\right)^r$

$E[Y] = \frac{pr}{1-p} \quad \text{var}(Y) = \frac{pr}{(1-p)^2}$

Dirichlet Distribution

$Y \sim \text{Dir}(\boldsymbol{\alpha})$

$y \in [0,1]; \sum y_i = 1 \quad a_i > 0$

$f(y|\boldsymbol{\alpha}) = \frac{\Gamma(\sum_{i=1}^K \alpha_i)}{\prod_{i=1}^K \Gamma(\alpha_i)} \prod_{i=1}^K y_i^{\alpha_i - 1}$

$E[Y_i] = \frac{\alpha_i}{\sum_k \alpha_k} \quad \text{var}(Y_i) = \frac{\alpha_i(\alpha_0 - \alpha_i)}{\alpha_0^2(\alpha_0+1)}; \alpha_0 = \sum_{i=1}^K \alpha_i$

$\text{cov}(Y_i, Y_j) = \frac{-\alpha_i\alpha_j}{\alpha_0^2(\alpha_0+1)}$

Multinomial Distribution

$Y \sim \text{Multinomial}(\boldsymbol{p})$

$\boldsymbol{y} \in \mathbb{N}^k \quad \boldsymbol{p} \in (0,1)^k, \sum p_i = 1$

$p(\boldsymbol{y}|(\boldsymbol{p}) = \frac{n!}{y_1!\dots y_k!} p_1^{y_1} \dots p_k^{y_k} \quad \phi(t;\boldsymbol{p}) = \left(\sum_{j=1}^k p_j e^{it_j}\right)^n$

$E[Y_i] = np_i \quad \text{var}(Y_i) = np_i(1-p_i) \quad \text{cov}(Y_i, Y_j) = -np_ip_j$

Geometric Distribution

$Y \sim \text{Geometric}(p)$

$y \in \mathbb{N} \quad p \in (0,1]$

$p(y|p) = (1-p)^{y-1}p \quad \phi(t;p) = \frac{pe^{it}}{1-(1-p)e^{it}}$

$E[Y] = \frac{1}{p} \quad \text{var}(Y) = \frac{1-p}{p^2}$

Cauchy Distribution

$Y \sim \text{Cauchy}(\mu, \gamma)$

$y \in (-\infty, \infty) \quad \mu \in \mathbb{R} \quad \gamma > 0$

$f(y|\mu,\gamma) = \frac{1}{\pi\gamma\left[1+\left(\frac{y-\mu}{\gamma}\right)\right]} \quad \phi(t;\mu,\gamma) = \exp\{\mu it - \gamma|t|\}$

$E[Y] = undefined \quad \text{var}(Y) = undefined \quad \text{median} = \mu$

F Distribution

$X \sim F(n_1, n_2)$

$n_1, n_2 > 0 \quad x \geq 0$

$f(x;n_1,n_2) = \frac{\sqrt{\frac{(n_1x)^{n_1} n_2^{n_2}}{(n_1x+n_2)^{n_1+n_2}}}}{xB(n_1/2, n_2/2)}$

$E[X] = \frac{n_2}{n_2-2} \quad \text{var}(X) = \frac{2n_2^2(n_1+n_2-2)}{n_1(n_2-2)^2(n_2-4)}$

$F(n_1, n_2) = \frac{\chi^2_{n_1}/n_1}{\chi^2_{n_2}/n_2}$

$F(n_1, n_2) = \frac{s_1^2}{\sigma_1^2} / \frac{s_2^2}{\sigma_2^2}$